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By-Rahmlow, Harold F.; And Others
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This programed mathematics textbook is for student use in vocational education courses. It was developed as part of a programed series covering 21 mathematical competencies which were identified by university researchers through task analysis of several occupational clusters. The development of a sequential content structure was also based on these mathematics competencies. After completion of this program the student should be able to: (1) know that "difference" indicates the operation of subtraction, (2) order any set of fractions, (3) subtract a small fraction of the form a/b where 0 is less than (a,b) when these are less than 100 from a larger fraction with the same denominator, (4) subtract a small fraction of the form a/b, where 0 is less than (a,b) and these are less than 100, from a larger fraction of the same form with unlike denominators, (5) subtract two literal fractions with common denominators, (6) subtract two literal fractions with unlike denominators, and (7) subtract a small mixed number from a larger one of the form Xa/b where 0 is less than (X,a,b) when these are less than 100. The material is to be used by individual students under teacher supervision. Twenty-six other programed texts and an introductory volume are available as VT 006 882-VT 006 909, and VT 006 975. (EM)

FINAL REPORT
Project No. 0E7-0031
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Report No. 16-F

Occupational Mathematics

SUBTRACTION OF FRACTIONS

June 1968

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U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
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Occupational Mathematics

SUBTRACTION OF FRACTIONS

Project No. 0E7-0031
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Report No. 16-F

by
Harold F. Rahmlow
Karl Ostheller
Clarence Potratz
Leonard T. Winchell
Arthur Snoey

June 1968

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Washington State University, Department of Education, Pullman, Washington
State Coordinating Council for Occupational Education, Olympia, Washington

OBJECTIVES

1. The student should know that the "difference" indicates the operation of subtraction.

2. The student should be able to order any set of fractions.
3. The student should be able to subtract a small fraction of the form a/b where $0<(a,b)<100$ from a larger fraction of this form with the same denominator.
4. The student should be able to subtract a small fraction of the form a/b, where $0<(a,b)<100$ from a larger fraction of this same form with unlike denominators.
5. The student should be able to subtract two literal fractions with common denominators.
6. The student should be able to subtract two literal fractions with unlike denominators.
7. The student should be able to subtract a small mixed number from a larger one of the form X a/b where $0<(x,a,b)<100$.

Greetings! You are about to begin improving your knowledge of basic mathematics. There are many important uses for the mathematics you are learning.

This booklet is not like your ordinary books. It is designed to help you learn as an individual. On the following pages you will find some information about mathematics. After the information is presented, you will be asked a question. Your answers to these questions will determine how you proceed through this booklet. When you have selected your answer to the question, turn to the page you are told to.

Do not write in this booklet. You may wish to have a pencil and some paper handy so you can write when you want to.

Remember this is not an ordinary book.

1. Study the material on the page.
2. Read the question on the page (you may want to restudy the material on the page).
3. Select the answer you believe is correct.
4. Turn to the page indicated by your answer.

Are you ready to begin?

(a) Yes — Turn to page 1

(b) No — Turn to page C

(c) HELP — Go see your teacher

Your answer was (b) No.

Well, this booklet is a little different.

Go back and read page B again. After you have read it, you will probably be ready to begin.

Hello! You are about to begin a unit on subtraction of fractions. The methods that will be used are very similar to those used in the unit on addition of fractions. If you are not familiar with how to add fractions, it would help you to take Unit 5A first. Otherwise, let's get started.

Turn to page 2.

Subtraction is one of the most basic and useful of the arithmetic operations. It would be nice if we only had to work with whole numbers, but many times we need to be able to subtract fractions.

For example, in order to build a certain model, you need 4 1/2 feet of wood. But you only have 1 1/3 feet. How much more do you need to buy? The solution of this problem would involve subtracting fractions.

Or perhaps a jar can hold 7/8 of a gallon of water. But you have already used 1/2 gallon of water. To find the amount remaining would require a knowledge of how to subtract fractions. Let's get started so you can learn how to solve such problems.

What is the difference of 7 and 5?

(a) I don't know what difference means
Turn to page 4

(b) 2 Turn to page 3

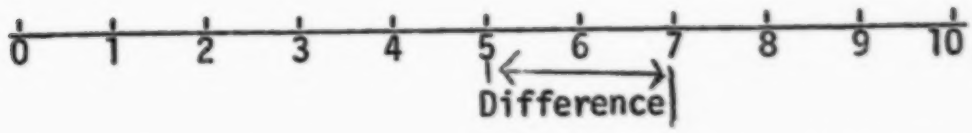
(c) 12 Turn to page 5

Fine! Remember that the difference of two numbers always means to subtract the smaller from the larger.

What is the difference between 7 and 13?

(a)	6	Turn to page 8
(b)	-6	Turn to page 6
(c)	20	Turn to page 12

The word <u>difference</u> is an important one to remember in mathematics. Let's look at a number line.



Each interval is one unit long. What is the difference between 7 and 5? You can see from the drawing that there are two units separating the 7 and the 5. So the difference between 7 and 5 is 7 - 5, or 2. To find the difference between any two numbers, always <u>subtract the smaller from the larger</u>.

Notice that the difference between 7 and 5 is the same as the difference between 5 and 7. It doesn't matter which order the numbers come; you still subtract the smaller from the larger. The difference is still 2.

Find the difference between 9 and 4.

(a) 36 — Turn to page 12

(b) 13 — Turn to page 6

(c) 5 — Turn to page 3

No! 12 is the sum of 7 and 5. You added the numbers.

This is incorrect.

Turn to page 4.

No. You missed the point on the last page. Let's try again. Difference means subtract. To find the difference between any two numbers, simply subtract the smaller from the larger. For example, the difference of 6 and 10 is 4 because 10 - 6 = 4.

What is the difference between 28 and 20?

(a) 18 Turn to page 17

(b) 8 Turn to page 3

(c) 10 Turn to page 15

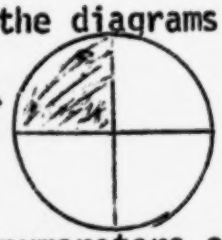
No! You still don't have it. Go back to page 8 and restudy it very carefully.

Excellent! Let's continue.

So far, we have stressed subtracting the smaller number from the larger. This is also true when subtracting fractions. In order to do this, you must first be able to tell which of two fractions is smaller. This is easy with whole numbers. It isn't quite as easy with fractions.

Recall that the denominator of a fraction tells how many equal parts there are in the whole. Therefore, 1/8 is smaller than 1/4 because there are 8 parts in the whole as compared to 4 parts. You can clearly see from the diagrams that 1/8 is smaller than 1/4.

1/4 ->



1/8 -->

When the numerators of fractions are the same, the fractions with the largest denominator will be the smallest.

(continued on next page)

Which of the following lists of fractions is correctly ordered from <u>smallest</u> <u>to largest</u>?

(a) 2/7, 2/9, 2/17, 2/21 Turn to page 10

(b) 4/15, 4/21, 4/17, 4/11, 4/7 Turn to page 13

(c) 1/25, 1/13, 1/9, 1/5, 1/3 Turn to page 11

No. You chose the wrong list. The correct list was (c) 1/15, 4/15, 7/15, 11/15, 13/15. Do you see why? Notice that the numerators are arranged in order from low to high. When the denominators are the same, you don't need to worry about them. Simply order the numerators. Certainly 4/15 is larger than 1/15; 7/15 is larger than 4/15, and so on. Got it? If not, re-read this page.

O.K. Let's try another problem.

Which of the following lists is correctly ordered from the lowest fraction to the highest?

(a) 7/5, 8/5, 9/5, 14/5 Turn to page 16

(b) 5/9, 8/9, 7/9, 10/9 Turn to page 21

(c) 13/16, 9/16, 7/16, 3/16 Turn to page 24

No. The list you picked was incorrect. In all three cases, the numerators did not change. So you were concerned only with the denominators.

Remember the larger the denominator, the smaller the fraction. The correct list from small to large was (c) 1/25, 1/13, 1/9, 1/5, 1/3.

Let's try again.

Which of the following is correctly ordered from smallest to largest?

(a) 3/5, 3/8, 3/10 Turn to page 14

(b) 3/10, 3/8, 3/5 Turn to page 11

(c) 3/8, 3/5, 3/10 Turn to page 7

Very good! You chose the correct list. In the last problem the numerators in each list were constant. This will not always be true.

If the denominators are constant, it is even easier. In that case, the fractions can be ordered simply by ordering the numerators. The fractions with smaller numerators will be the smaller fractions.

Let's try one.

Which of the following lists of fractions is correctly ordered from smallest to largest?

(a) 1/9, 3/9, 2/9, 5/9, 4/9 Turn to page 9

(b) 3/17, 6/17, 5/17, 4/17, 2/17 Turn to page 18

(c) 1/15, 4/15, 7/15, 11/15, 13/15 Turn to page 16

No. You missed the point on the last page. Let's try again. Difference means subtract. To find the difference between any two numbers, simply subtract the smaller from the larger.

For example, the difference of 6 and 10 is 4 because 10 - 6 = 4.

What is the difference between 28 and 20?

(a) 18 Turn to page 17

(b) 8 Turn to page 3

(c) 10 Turn to page 15

No. The list you picked was incorrect. In all three cases, the numerators did not change. So you were concerned only with the denominators.

Remember, the larger the denominator, the smaller the fraction. The correct list from small to large was (c) 1/25, 1/13, 1/9, 1/5, 1/3.

Let's try again.

Which of the following is correctly ordered from smallest to largest?

(a) 3/5, 3/8, 3/10 Turn to page 14

(b) 3/10, 3/8, 3/5 Turn to page 11

(c) 3/8, 3/5, 3/10 Turn to page 7

No. You still don't have it. Go back to page 8 and restudy it very carefully.

No. You still haven't caught on. Go back to page 4 and read very carefully this time.

Fine. We have now looked at the case where the numerator stays constant and the case where the denominator stays constant. But what about when neither stays constant? There is an easy answer. Let's look at an example.

Which of the fractions 3/7 and 5/9 is smaller? This isn't quite the same as the other two cases. The best way to solve it is to first find a common denominator.

For our problem we have 3/7 and 5/9. Our common denominator is the product of the two denominators, namely 9 x 7 or 63.

Now we need to change 3/7 and 5/9 to equivalent fractions with 63 as denominator. Thus:

$$3/7 = 3/7 \cdot 9/9 = 27/63$$

$$5/9 = 5/9 \cdot 7/7 = 35/63$$

Continued on next page

We now compare 27/63 with 35/63. It is easy to see that 27/63 is smaller than 35/63. Since these fractions are equivalent to the original ones, 3/7 is smaller than 5/9.

You try one now.

Which fraction is <u>larger</u>--3/5 or 5/8?

(a) 3/5 Turn to page 19

(b) 5/8 Turn to page 20

No. You still haven't caught on. Go back to page 4 and read very carefully this time.

No. You chose the wrong list. The correct list was (c) 1/15, 4/15, 7/15, 11/15, 13/15. Do you see why? Notice that the numerators are arranged in order from low to high. When the denominators are the same, you don't need to worry about them. Simply order the numerators. Certainly 4/15 is larger than 1/15; 7/15 is larger than 4/15, and so on. Got it? If not, reread this page.

O.K. Let's try another problem.

Which of the following lists is correctly ordered from the lowest fraction to the highest?

(a) 7/5, 8/5, 9/5, 14/5 Turn to page 16

(b) 5/9, 8/9, 7/9, 10/9 Turn to page 21

(c) 13/16, 9/16, 7/16, 3/16 Turn to page 24

No. Not quite. Let's solve this one together. We had 3/5 and 5/8. To see which is larger, we first need a common denominator. 8 x 5 or 40 will be our common denominator.

Then 3/5 = 3/5 x 8/8 = 24/40

and 5/8 = 5/8 x 5/5 = 25/40.

Since 25/40 is larger than 24/40, we know that 5/8 is larger than 3/5 since we didn't change the original values.

Here's another one to try.

Which of the fractions 3/7 and 2/5 is <u>larger</u>?

(a) 3/7 Turn to page 20

(b) 2/5 Turn to page 25

Right. Very good!

We will now expand this idea. Sometimes you will have to order more than two fractions. The principle is still the same. Find a common denominator for all the fractions, and then put them in order by simply looking at their numerators.

Let's work a problem together. Put the following fractions in order from smallest to largest: 1/2, 3/4, 2/3. We first need a common denominator for the fractions 1/2, 3/4, and 2/3. 2 x 4 x 3 = 24 will be the common denominator.

$$1/2 = 1/2 \times 4/4 \times 3/3 = \frac{1 \times 4 \times 3}{2 \times 4 \times 3} = 12/24.$$

$$3/4 = 3/4 \times 2/2 \times 3/3 = \frac{3 \times 2 \times 3}{4 \times 2 \times 3} = 18/24.$$

$$2/3 = 2/3 \times 2/2 \times 4/4 = \frac{2 \times 2 \times 4}{3 \times 2 \times 4} = 16/24.$$

The correct sequence is then 12/24, 16/24, and 18/24. So the original fractions would be 1/2, 2/3, and 3/4.

Continued on next page

Are you ready to try one? Be sure you understand the material on the previous page first.

Arrange the fractions 3/5, 1/4 and 2/3 in order from smallest to largest.

(a) 1/4, 2/3, 3/5 Turn to page 27

(b) 1/4, 3/5, 2/3 Turn to page 23

(c) 3/5, 1/4, 2/3 Turn to page 29

No! You missed again. Come on now. It's easy. Let's go over it once more.

When the denominators are the same, all you need to do is arrange the numerators in order.

Which list is in correct order from smallest to largest?

(a) 1/7, 3/7, 2/7 Turn to page 22

(b) 4/9, 5/9, 8/9 Turn to page 28

(c) 5/13, 3/13, 1/13 Turn to page 26

Being able to arrange fractions in order is quite important to understand before going on. Go see if your teacher can help you in this area and then return to page 8.

Excellent! You seem to be getting it.

Now let's go on and learn how to subtract fractions. Subtraction is almost like addition. After you have a common denominator, you simply subtract the numerators, keeping the common denominator in the final answer. (Remember the symbol used for subtraction is "-"; i.e., 8 - 5 = 3.)

O.K. Here's one for you.

What is 3/5 - 1/5?

(a) 4/5 Turn to page 30

(b) 1/5 Turn to page 32

(c) 2/5 Turn to page 34

No! You missed again. Come on now. It's easy. Let's go over it once more.

When the denominators are the same, all you need to do is arrange the numerators in order.

Which list is in correct order from the smallest to largest?

(a) 1/7, 3/7, 2/7 Turn to page 22

(b) 4/9, 5/9, 8/9 Turn to page 28

(c) 5/13, 3/13, 1/13 Turn to page 26

No. These are a little tricky but are not hard if they are done correctly.

Go back to page 16 and study it very carefully before proceeding.

Being able to arrange fractions in order is quite important to understand before going on. Go see if your teacher can help you in this area and then return to page 8.

Your choice was incorrect. Let's work it.

The fractions were 3/5, 1/4, and 2/3. 5 x 4 x 3 can be our common denominator.

$3/5 = 3/5 \times 4/4 \times 3/3 = \frac{3 \times 4 \times 3}{5 \times 4 \times 3} = 36/60.$

$1/4 = 1/4 \times 5/5 \times 3/3 = \frac{1 \times 5 \times 3}{4 \times 5 \times 3} = 15/60.$

$2/3 = 2/3 \times 5/5 \times 4/4 = \frac{2 \times 5 \times 4}{3 \times 5 \times 4} = 40/60.$

The correct order is clearly 15/60, 36/60, and 40/60. The original fractions would then be 1/4, 3/5, and 2/3.

O.K. Here's another one.

Which fraction would belong in the <u>middle</u> if the fractions 1/3, 2/7, and 3/5 were ordered?

(a)	1/3	Turn to page 35
(b)	2/7	Turn to page 31
(c)	3/5	Turn to page 33

I think you have it. Here's one more.

Pick the list that is correctly ordered from low to high.

(a) 7/5, 8/5, 9/5, 14/5 Turn to page 16

(b) 5/9, 8/9, 7/9, 10/9 Turn to page 22

(c) 13/16, 9/16, 7/16, 3/16 Turn to page 26

Your choice was incorrect. Let's work it.

The fractions were 3/5, 1/4, and 2/3. 5 x 4 x 3 can be our common denominator.

$3/5 = 3/5 \times 4/4 \times 3/3 = \frac{3 \times 4 \times 3}{5 \times 4 \times 3} = 36/60.$

$1/4 = 1/4 \times 5/5 \times 3/3 = \frac{1 \times 5 \times 3}{4 \times 5 \times 3} = 15/60.$

$2/3 = 2/3 \times 5/5 \times 4/4 = \frac{2 \times 5 \times 4}{3 \times 5 \times 4} = 40/60.$

The correct order is clearly 15/60, 36/60, and 40/60. The original fractions would then be 1/4, 3/5, and 2/3.

O.K. Here's another one.

Which fraction would belong in the <u>middle</u> if the fractions 1/3, 2/7, and 3/5 were ordered?

(a)	1/3	Turn to page 35
(b)	2/7	Turn to page 31
(c)	3/5	Turn to page 33

No. Not quite. Let's go over it once more.

$3/5 - 1/5 = \frac{3 - 1}{5} = 2/5$. That's all there is to it.

Here's another one.

What is 6/7 - 2/7?

(a) 5/7 Turn to page 36

(b) 4/7 Turn to page 34

(c) 3/7 Turn to page 39

No! Your choice was incorrect. Here's a clue.

1/3 = 1/3 x 7/7 x 5/5 = ?

2/7 = 2/7 x 3/3 x 5/5 = ?

3/5 = 3/5 x 7/7 x 3/3 = ?

When you have completed the equations above, return to page 27 and try the problem again.

No. Not quite. Let's go over it once more.

$3/5 - 1/5 = \frac{3 - 1}{5} = 2/5$. That's all there is to it.

Here's another one.

What is 6/7 - 2/7?

(a) 5/7 Turn to page 36

(b) 4/7 Turn to page 34

(c) 3/7 Turn to page 39

No. Your choice was incorrect. Here's a clue.

1/3 = 1/3 x 7/7 x 5/5 = ?

2/7 = 2/7 x 3/3 x 5/5 = ?

3/5 = 3/5 x 7/7 x 3/3 = ?

When you have completed the equations above, return to page 27 and try the problem again.

Very good! Here's one with numbers a little larger.

What is 19/23 - 7/23?

(a) 12/23 Turn to page 42

(b) 23/12 Turn to page 37

(c) 26/23 Turn to page 40

Right! 1/3 is the correct choice.

Here's one more.

The fractions 2/3, 3/4, and 4/5 are arranged in the correct order from smallest to largest.

(a) True Turn to page 23

(b) False Turn to page 38

No. You're just not catching on.

Perhaps you should review how to subtract whole numbers. That's really all that is involved--subtracting integer numerators.

Go see your teacher for suggestions. Then return to page 23.

Oops! You missed this one.

Remember, another way to write 19/23 - 7/23 is $\frac{19 - 7}{23}$. That's not hard at all. The answer has to be 12/23.

What is 13/37 - 9/37?

(a) 22/37 Turn to page 36

(b) 4/37 Turn to page 41

(c) Neither of the above Turn to page 39

No. You don't seem to get it.

Go back to page 20 and start from there. Read the material very carefully this time.

No. You're just not catching on.

Perhaps you should review how to subtract whole numbers. That's really all that is involved--subtracting integer numerators.

Go see your teacher for suggestions. Then return to page 23.

Oops! You missed this one.

Remember, another way to write 19/23 - 7/23 is $\frac{19 - 7}{23}$. That's not hard at all. The answer has to be 12/23.

What is 13/37 - 9/37?

(a) 22/37 Turn to page 36

(b) 4/37 Turn to page 41

(c) Neither of the above Turn to page 39

Good! You got that one right. How about one more just to be sure? Let's try.

Subtract 17/29 from 28/29.

(a) 17/29 Turn to page 36

(b) 45/29 Turn to page 39

(c) 11/29 Turn to page 42

Fine! You have it.

Here's one that may be a little tricky. Think about it before you answer.

What is 7/10 - 3/10?

(a) 2/5 Turn to page 45

(b) 3/10 Turn to page 47

(c) Not given Turn to page 48

All the answers have the correct value. However, only one was reduced to simplest terms.

$$9/16 - 5/16 = \frac{9 - 5}{16} = 4/16 = \frac{\cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times 2 \times 2} = 1/4$$

Do you remember now how to reduce fractions? It should be done in every answer.

Reduce 42/54 to lowest terms.

(a)	21/27	Turn to page 50
(b)	14/18	Turn to page 53
(c)	7/9	Turn to page 46

You're correct.

Here's another one.

What is 5/12 - 2/7?

(a)	11/84	Turn to page 57
(b)	3/7	Turn to page 60
(c)	10/74	Turn to page 64

Good! You got that one.

Find the difference between 9/16 and 5/16. Make sure your answer is in simplest form.

(a) 4/16 Turn to page 43

(b) 1/4 Turn to page 51

(c) 2/8 Turn to page 49

Right! You may have it this time. Let's try another just to be sure.

Subtract 5/18 from 17/18. Express in lowest terms.

(a) 6/9 Turn to page 53

(b) 2/3 Turn to page 51

(c) 12/18 Turn to page 50

Oops. You slipped up there. Let's work it.

$7/10 - 3/10 = \frac{7 - 3}{10} = 4/10$

But $4/10 = \frac{2 \cdot 2}{2 \cdot 5} = 2/5.$

Don't forget to reduce your answers whenever you can.

Do you remember how?

Reduce the fraction 60/84 to lowest terms.

(a)	30/42	Turn to page 50
(b)	20/28	Turn to page 53
(c)	5/7	Turn to page 45

Oops. You slipped up there. Let's work it.

$$7/10 - 3/10 = \frac{7 - 3}{10} = 4/10.$$

But

$$4/10 = \frac{2 \cdot 2}{2 \cdot 5} = 2/5.$$

Don't forget to reduce your answers whenever you can.

Do you remember how?

Reduce the fraction 60/84 to lowest terms.

(a)	30/42	Turn to page 50
(b)	20/28	Turn to page 53
(c)	5/7	Turn to page 45

All the answers have the correct value. However, only one was reduced to simplest terms.

$$9/16 - 5/16 = \frac{9 - 5}{16} = 4/16 = \frac{\cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times 2 \times 2} = 1/4.$$

Do you remember now how to reduce fractions? It should be done in every answer.

Reduce 42/54 to lowest terms.

(a) 21/27 Turn to page 50

(b) 14/18 Turn to page 53

(c) 7/9 Turn to page 46

No. The fraction you chose is not in lowest terms.

You need more work on this. Go to page 20 of Unit 3.

When you have learned how to reduce fractions,

return to page 42 of this Unit.

Very good! Let's go on.

You remember that in order to subtract fractions there must always be a common denominator.

See if you can do this one correctly.

What is 2/3 - 1/4?

(a) 1/4 Turn to page 54

(b) 5/12 Turn to page 44

(c) 1/2 Turn to page 56

No. You seem to be having difficulty with common denominators.

Go to page 29 of Unit 5A to help you with this.

Then return to page 51 of this Unit.

No. The fraction you chose is not in lowest terms.

You need more work on this.

Go to page 20 of Unit 3.

When you have learned how to reduce fractions,

return to page 42 of this Unit.

No. That's not it. Let's go over how to do one that needs a common denominator.

We were subtracting 2/3 - 1/4. Our common denominator will be the product of the existing denominators; namely, 3 x 4 or 12.

Then 2/3 = 2/3 x 4/4 = 8/12

and 1/4 = 1/4 x 3/3 = 3/12

So, 2/3 - 1/4 = 8/12 - 3/12 = $\frac{8 - 3}{12}$ = 5/12.

Study the solution carefully before going on.

Now find 5/6 - 3/4.

(a) 1/3 Turn to page 52

(b) 1/6 Turn to page 58

(c) 1/12 Turn to page 44

Come on. You can do better than that.

In the last problem a was the common denominator for both fractions. So $3/a - 2/a = \frac{3 - 2}{a} = 1/a$.
Simple, huh?

O.K., then, see if you can do better this time.

What is 13/4K - 2/4K?

(a) 11/4K Turn to page 62

(b) 11/K Turn to page 68

(c) 11K Turn to page 66

No. That's not it. Let's go over how to do one that needs a common denominator. We were subtracting 2/3 - 1/4. Our common denominator will be the product of the existing denominators; namely, 3 x 4 or 12.

Then 2/3 = 2/3 x 4/4 = 8/12

and 1/4 = 1/4 x 3/3 = 3/12.

So $2/3 - 1/4 = 8/12 - 3/12 = \frac{8 - 3}{12} = 5/12$.

Study the solution carefully before going on.

Now find 5/6 - 3/4.

(a)	1/3	Turn to page 52
(b)	1/6	Turn to page 58
(c)	1/12	Turn to page 44

Excellent! You can now subtract numeric fractions. (Fractions that contain only numbers). Everything we have done with numbers also applies to the use of letters. Keeping this in mind, let's see if you can do one.

What is 3/a - 2/a?

(a) 1/a Turn to page 62

(b) 1a Turn to page 55

(c) 1/2a Turn to page 59

No. You seem to be having difficulty with common denominators.

Go to page 29 of Unit 5A to help you with this.

Then return to page 51 of this Unit.

Come on. You can do better than that.

In the last problem a was the common denominator for both fractions. So $3/a - 2/a = \frac{3 - 2}{a} = 1/a$. Simple, huh?

O.K., then see if you can do better this time.

What is 13/4K - 2/4K?

(a) 11/4K Turn to page 62

(b) 11/K Turn to page 68

(c) 11K Turn to page 66

Incorrect. It's not really much different from the last one.

We must first change 5/12 and 2/7 to equivalent fractions with a common denominator. 12 x 7 or 84 will be the common denominator. Then:

$5/12 = 5/12 \times 7/7 = 35/84$

and $2/7 = 2/7 \times 12/12 = 24/84$.

So, $5/12 - 2/7 = 35/84 - 24/84 = \frac{35 - 24}{84} = 11/84$.

Let's try another one.

What is 5/8 - 1/3?

(a) 4/12 Turn to page 58

(b) 1/3 Turn to page 52

(c) 7/24 Turn to page 65

Watch it! You got trapped on that one.

Remember you must put both fractions over the common denominator before you can start the subtraction. Do not merely subtract numerators.

O.K. Let's try again.

What is 15/M - 8/N?

(a) (15N - 8M)/MN Turn to page 69

(b) 7/MN Turn to page 66

(c) Not Given Turn to page 68

Good! It's really no harder than working with numbers.

Find: a/x - b/x.

(a) -ab/x Turn to page 75

(b) a - b/x Turn to page 71

(c) (a-b)/x Turn to page 69

Good! That's correct.

So far, we have been dealing only with simple fractions. Let's take a look at mixed numbers now. Here's an example.

Subtract 2 1/3 from 4 2/3. To work this one, it is easier to write it as:

$$\begin{array}{r} 4\ 2/3 \\ -1\ 1/3 \\ \hline 3\ 1/3 \end{array} \leftarrow \text{--- Answer}$$

All you do is subtract the fraction from the fraction and the whole number from the whole number.

Now you try it.

What is 6 5/7 - 2 2/7?

(a) 3 4/7 Turn to page 73

(b) 4 3/7 Turn to page 70

(c) 2 6/7 Turn to page 78

Incorrect. It's not really much different from the last one.

We must first change 5/12 and 2/7 to equivalent fractions with a common denominator. 12 x 7 or 84 will be the common denominator. Then:

$5/12 = 5/12 \times 7/7 = 35/84$

and $2/7 = 2/7 \times 12/12 = 24/84$.

So $5/12 - 2/7 = 35/84 - 24/84 = \frac{35 - 24}{84} = 11/84$.

Let's try another one.

What is 5/8 - 1/3?

(a) 4/12 Turn to page 58

(b) 1/3 Turn to page 52

(c) 7/24 Turn to page 65

Right! That's good. Can you do another one now?

Find: 7/13 - 2/7 = ?

(a)	5/13	Turn to page 52
(b)	23/91	Turn to page 57
(c)	17/81	Turn to page 58

No. You're making it much too difficult.

Go see your teacher for help with fractions containing letters in the denominator.

Then return to page 57 of this Unit.

Incorrect. Let's work it.

Remember the problem was to find $3/x - 4/y$. What do you do first? Find a common denominator. Right? O.K. xy is our common denominator. So:

$$3/x = 3/x \cdot y/y = 3y/xy$$

and $4/y = 4/y \cdot x/x = 4x/xy$

Then, $3/x - 4/y = 3y/xy - 4x/xy = (3y-4x)/xy$.

Study this page until you're sure you understand it.

What is $7/v - 2/w$?

(a) $5/vw$ Turn to page 61

(b) $7w/2v$ Turn to page 66

(c) $(7w - 2v)/vw$ Turn to page 69

No. You're making it much too difficult.

Go see your teacher for help with fractions containing letters in the denominator.

Then return to page 57 of this Unit.

Very good! Your answer was correct.

Here's one using two different letters as denominators. Don't get thrown by it. Use exactly the same procedure as you have all along.

What is $3/x - 4/y$?

(a) $-1/xy$ Turn to page 67

(b) $(3y - 4x)/xy$ Turn to page 63

(c) $4x/3y$ Turn to page 72

Very good! You got that one.

If the denominators of the fractions are not alike, you must change them to equivalent fractions with a common denominator before subtracting--just like you always have. Don't forget to reduce all answers.

What is 4 5/6 - 1 1/3?

(a) 3 2/3 Turn to page 74

(b) 3 1/3 Turn to page 77

(c) 3 1/2 Turn to page 79

No. Read this explanation carefully.

X was the common denominator, and the numerators were letters also. There is not much you can do with this type. Just remember to group the new numerator inside parenthesis. In our problem, it was merely $a/x - b/x = (a - b)/x$.

O.K. Try this one.

What is $3x/b - 2y/b$?

(a) xy/b Turn to page 68

(b) $(3x - 2y)/b$ Turn to page 69

(c) $(3x - 2y)/2b$ Turn to page 66

Incorrect. Let's work it.

Remember the problem was to find $3/x - 4/y$. What do you do first? Find a common denominator. Right? O.K. xy is our common denominator. So:

$$3/x = 3/x \cdot y/y = 3y/xy$$

and $4/y = 4/y \cdot x/x = 4x/xy$

Then $3/x - 4/y = 3y/xy = 4x/xy = (3y - 4x)/xy$.

Study this page until you're sure you understand it.

What is $7/v - 2/w$?

(a) $5/vw$ Turn to page 61

(b) $7w/2v$ Turn to page 66

(c) $(7w - 2v)/vw$ Turn to page 69

Come on now. You didn't read the last page carefully. It's really easy.

$$\begin{array}{r} 6\ 5/7 \\ -\ 2\ 2/7 \\ \hline 4\ 3/7 \end{array} \leftarrow \text{Answer}$$

That's all there is to it. Subtract the whole numbers and subtract the fractions.

What is 3 4/5 - 1 3/5?

(a) 2 1/5 Turn to page 79

(b) 1 4/5 Turn to page 81

(c) 2 2/5 Turn to page 84

Incorrect. Let's work it together.

4 5/6 - 1 1/3 was the problem. Notice that 6 can be a common denominator since both denominators are multiples of 6. (This means that both 6 and 3 will divide into 6 evenly.)

$$1/3 = 1/3 \times 2/2 = 2/6$$

Our problem will then be

$$\begin{array}{r} 4\ 5/6 \\ -1\ 2/6 \\ \hline 3\ 3/6 \end{array}$$

But, 3/6 = 1/2 x 3/3 = 1/2. So the final reduced answer is 3 1/2.

Don't let reducing the answer fool you. This happens very often.

All right. Let's try again.

What is 6 11/12 - 1 1/4?

(a) 5 5/6 Turn to page 83

(b) 5 2/3 Turn to page 85

(c) 4 7/12 Turn to page 87

No. Read this explanation carefully.

X was the common denominator, and the numerators were letters also. There is not much you can do with this type. Just remember to group the new numerator inside parenthesis. In our problem, it was merely $a/x - b/x = (a - b)/x$.

O.K. Try this one.

What is $3x/b - 2y/b$?

(a) wy/b Turn to page 68

(b) $(3x - 2y)/b$ Turn to page 69

(c) $(3x - 2y)/2b$ Turn to page 66

No. Your choice was incorrect.

Here's the problem in detail. The problem was 6 2/3 - 2 3/4. He will use 3 x 4 or 12 as the common denominator.

Then 2/3 = 2/3 x 4/4 = 8/12, So 6 2/3 = 6 8/12
and 3/4 = 3/4 x 3/3 = 9/12 which makes 2 3/4 = 2 9/12.

Also 6 8/12 = 6 + 8/12 = (5 + 12/12) + 8/12
= 5 + (12/12 + 8/12)
= 5 + 20/12
= 5 20/12

Now we can subtract
$$\begin{array}{r} 5\ 20/12 \\ -\ 2\ \ 9/12 \\ \hline 3\ 11/12 \end{array}$$ <--- Answer

Does that help any? Study this problem carefully.

Let's try again.

What is 6 1/3 - 4 3/5?

(a) 2 4/15 Turn to page 89

(b) 2 1/15 Turn to page 86

(c) 1 11/15 Turn to page 88

Incorrect. Let's work it together.

4 5/6 - 1 1/3 was the problem. Notice that 6 can be a common denominator since both denominators are multiples of 6. (This means that both 6 and 3 will divide into 6 evenly.) 1/3 = 1/3 x 2/2 = 2/6.

Our problem will then be

$$\begin{array}{r} 4\ 5/6 \\ -1\ 2/6 \\ \hline 3\ 3/6 \end{array}$$

But 3/6 = 1/2 x 3/3 = 1/2. So the final reduced answer is 3 1/2. Don't let reducing the answer fool you. This happens very often.

All right. Let's try again.

What is 6 11/12 - 1 1/4?

(a) 5 5/6 Turn to page 83

(b) 5 2/3 Turn to page 85

(c) 4 7/12 Turn to page 87

Come on now. You didn't read the last page carefully. It's really easy.

$$\begin{array}{r} 6\ 5/7 \\ -\ 2\ 2/7 \\ \hline 4\ 3/7 \end{array}$$

That's all there is to it. Subtract the whole numbers and subtract the fractions.

What is 3 4/5 - 1 3/5?

(a) 2 1/5 Turn to page 79

(b) 1 4/5 Turn to page 81

(c) 2 2/5 Turn to page 84

Excellent! Let's go one step further.

For example, subtract 2 3/5 from 6 1/3. Finding

5 x 3 = 15 for a common denominator, we have

3/5 = 3/5 x 3/3 = 9/15, so 2 3/5 = 2 9/15

and

1/3 = 1/3 x 5/5 = 5/15 which makes 6 1/3 = 6 5/15.

Our problem is now

$$\begin{array}{r} 6\ 5/15 \\ -2\ 9/15 \\ \hline \end{array}$$

Clearly 9/15 is larger than 5/15. What do we do?

Go on to page 80.

You should recall that 6 5/15 really means 6 + 5/15. This can be expressed in other equivalent forms if necessary. Since we need more 15ths in order to subtract and we know 15/15 = 1, let 6 = 5 + 15/15.

O.K.? Then we can say:

$$6\ 5/15 = 5 + 15/15 + 5/15$$
$$= 5 + 20/15$$
$$= 5\ 20/15$$

You should be familiar with this from addition of fractions.

Now this problem is easy.

$$\begin{array}{r} 5\ 20/15 \\ -2\ \ 9/15 \\ \hline 3\ 11/15 \end{array} \leftarrow \text{--- Answer}$$

Be sure to reduce your final answer. In this example it is already reduced.

Study this example carefully before going on. O.K.

Now you find 6 2/3 - 2 3/4.

(a) 3 11/12 — Turn to page 88

(b) 4 3/4 — Turn to page 76

(c) 3 1/3 — Turn to page 82

Come on, now. It's not that difficult. You didn't study page 63 very well.

Go back to page 63 and proceed from there.

No. Your choice was incorrect. Here's the solution in detail.

The problem was 6 2/3 - 2 3/4. We will use 3 x 4 or 12 as the common denominator. Then 2/3 = 2/3 x 4/4 = 8/12 so 6 2/3 = 6 8/12, and 3/4 = 3/4 x 3/3 = 9/12 which makes 2 3/4 = 2 9/12.

Also 6 8/12 = 6 + 8/12 = (5 + 12/12) + 8/12

= 5 + (12/12 + 8/12)

= 5 + 20/12

= 5 20/12

Now we can subtract 5 20/12
-2 9/12
3 11/12 <--- Answer

Does that help any? Study this problem carefully.

Let's try again.

What is 6 1/3 - 4 3/5?

(a) 2 4/15 Turn to page 89

(b) 2 1/15 Turn to page 86

(c) 1 11/15 Turn to page 88

No. You didn't study page 74 enough.

Return to page 74 and carefully study the problem worked there. Then try again.

Come on, now. It's not that difficult.

You didn't study page 63 very well. Go back to page 63 and proceed from there.

Good! You got that one. Let's try another problem similar to it.

What is 5 2/3 - 4 7/15?

(a) $1\frac{1}{5}$ Turn to page 79

(b) 14/15 Turn to page 81

(c) $1\frac{1}{3}$ Turn to page 84

These problems are a little harder than most of them.

Go see your teacher for further explanation and then return to page 79 of this Unit.

No. You didn't study page 74 enough.

Return to page 74 and <u>carefully</u> study the problem worked there. Then try again.

Congratulations! You have completed this Unit on subtraction of fractions. Perhaps a quick review of the material covered would be helpful.

1. The difference between any two numbers is found by subtracting the smaller from the larger.
2. Ordering of Fractions
   A. If the numerators are the same, the larger the denominator the smaller the fraction.
   B. If the denominators are the same, the smaller the numerator the smaller the fraction.
   C. When neither numerators or denominators are the same, find common denominator and use the rules A and B above.
3. To subtract fractions with the same denominator, simply subtract numerators and keep the common denominator.
4. To subtract fractions with different denominators, find a common denominator and subtract as above.
5. The rules for subtraction of fractions with numbers applies to fractions with letters as well.
6. To subtract mixed numbers, work with the whole numbers and the fractions separately.
7. Be sure to reduce your answer whenever possible.

You are now ready for a test on this material. Go tell your teacher you have completed this unit.

These problems are a little harder than most of them.

Go see your teacher for further explanation and then return to page 79 of this Unit.

NORTHWEST REGIONAL EDUCATIONAL LABORATORY
400 Lindsay Building 710 S. W. Second Avenue
Portland, Oregon 97204

CAI MATHEMATICS

TEST QUESTIONS

UNIT 5B - SUBTRACTION OF FRACTIONS

Directions: The correct answers will always be expressed in lowest terms.

1. What is the difference between 5/14 and 1/14?

   a) 5/0

   b) 4/14

   c) 2/7

2. What is the correct order of the fractions 1/3, 2/7 and 3/5 from smallest to largest?

   a) 1/3, 2/7, 3/5

   b) 2/7, 1/3, 3/5

   c) 1/3, 3/5, 2/7

3. What is the difference between 9/17 and 3/17?

   a) 6/17

   b) 12/17

   c) 6/0

4. Subtract 1/4 from 5/8

   a) 4/4 (or 1)

   b) 3/8

   c) You can't because 5/8 is larger than 1/4

5. What is K/X - F/X?

   a) (K-F)/0

   b) $\frac{K - F}{X}$

   c) KF/X

Unit 5B (continued)

6. Subtract 5/x from 4/y

   a) -1/xy

   b) 1/(x-y)

   c) $\frac{4x - 5y}{xy}$

7. What is 3 1/2-2 1/4?

   a) 1 1/4

   b) 7/4

   c) 1/4

8. Order the fractions $\frac{2}{3}$, $\frac{1}{5}$, $\frac{4}{15}$, from smallest to largest

   a) 1/5, 4/15, 2/3

   b) 4/15, 2/3, 1/5

   c) 1/15, 2/3, 4/15

9. Subtract 3/10 from 9/10

   a) 3/10

   b) 3/5

   c) 12/20

10. What is the difference between 11/20 and 2/5?

   a) 3/20

   b) 9/15

   c) 3/5

11. What is the difference between 13/k and 8/k?

   a) 21/k

   b) 5/2k

   c) 5/k

Unit 5B (continued)

12. $\frac{M}{X} - \frac{N}{K} =$

    a) $\frac{MK - NX}{XK}$

    b) $\frac{M - N}{X - K}$

    c) $\frac{M - N}{XK}$

13. Subtract: 5 2/3 - 4 7/15

    a) 1 1/5

    b) 14/15

    c) 1 1/3

14. If the fractions 3/4, 7/8 and 13/16 were ordered from smallest to largest, which term should be in the middle?

    a) 3/4

    b) 7/8

    c) 13/16

15. What is $\frac{4}{5} - \frac{1}{5}$?

    a) 2/5

    b) 3/5

    c) 4/5

16. 7/15 - 2/5 =

    a) 1/15

    b) 1/2

    c) 9/20

1 . What is A/B - MX/B?

    a) $\frac{AMX}{B}$

    b) $\frac{A-MX}{B}$

    c) $\frac{A-M-X}{B}$

Unit 5B (continued)

18. What is 14/c - 14/d?

a) 0

b) $\frac{14cd}{cd}$

c) $\frac{14d-14c}{cd}$

19. 5 3/4 - 2 1/12 =

a) 3 1/8

b) 3 2/3

c) 3 1/3

20. The fractions 1/3, 1/2, and 7/12 are in the correct order from smallest to largest

a) True

b) False

21. Subtract: 15/11 - 7/11 =

a) 6/11

b) 8/0

c) Not given

22. 15/18 - 1/2 =

a) 6/18

b) 1/3

c) 7/8

23 5/y - K/y =

a) 5k/2y

b) $\frac{5 - k}{2y}$

c) $\frac{5 - k}{y}$

Unit 5B (continued)

24. Subtract: $8/k - 1/r$

a) $\frac{8r - k}{kr}$

b) $\frac{7}{kr}$

c) $\frac{7}{k - r}$

25. What is 13 2/3 - 4 1/4?

a) 9 5/12

b) 9 1/3

c) 10 1/12

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400 Lindsay Building 710 S. W. Second Avenue
Portland, Oregon 97204

CAI MATHEMATICS

ANSWER SHEET

UNIT 5B - SUBTRACTION OF FRACTIONS

1. c
2. b
3. a
4. b
5. b
6. c
7. a
8. a
9. b
10. a
11. c
12. a
13. a
14. c
15. b
16. a
17. b
18. c
19. b
20. a
21. c
22. b
23. c
24. a
25. a

Objectives	Questions
1	1, 3, 10, 11
2	2, 8, 14, 20
3	3, 9, 15, 21
4	4, 10, 16, 22
5	5, 11, 17, 23
6	6, 12, 18, 24
7	7, 13, 19, 25

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OFFICE OF EDUCATION

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TITLE

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PERSONAL AUTHOR(S)

Rahmlow, Harold; Ostheller, Karl; Potratz, Clarence; Winchell, L.; Snoey, Arthur

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ABSTRACT

One book of a 21-book series of programmed instruction materials designed to help pupils acquire mathematics capabilities most useful in sub-professional level occupations. Other programmed books in the series are:

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Representing Numbers by Letters
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Concepts of Decimals and Fractions
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Multiplication of Decimals
Division of Decimals
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